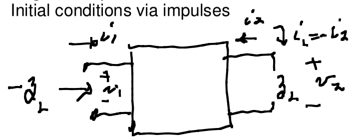
Lapace transform inversions
Partial fractions
Negative impedance converters
Initial conditions via impulses

Exponential of a square matrix
Singular value decompositions
Loaded Richards' sections



INIC = current negative impedance converter
VNIC = voltage NIC

if $v_2 = v_1$, $i_1 = i_2$ INIC

$v_2 = z_L i_L = -z_L i_2$
$v_1 = -z_L i_1$

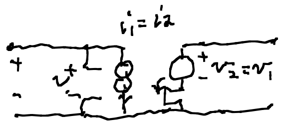
$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

if $v_2 = -v_1$, $i_1 = -i_2$ VNIC

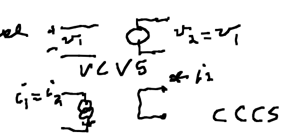
$v_2 = z_L(-i_2) = z_L i_1$
$-v_1 = z_L i_1$

$$\begin{bmatrix} +1 & +1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

no Z or Y matrices   det A = det B if are written as $Av = Bi$



use



Bilateral Laplace transform

$$\mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-st}\, dt \quad \text{depends on } s \text{ for convergence}$$

$$F(s) = \frac{3s}{(s+1)(s+2)} = \frac{k_1}{(s+1)} + \frac{k_2}{(s+2)}$$

$$= \frac{-3}{s+1} + \frac{6}{s+2}$$

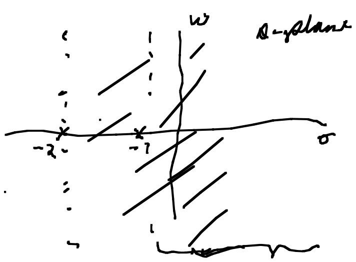
$k_1 = \left.\frac{3s}{s+2}\right|_{s=-1} = \frac{-3}{-1+2} = -3$

multiply by $s+1$

$$\left.\frac{3s(s+1)}{(s+1)(s+2)} = \frac{3s}{s+2}\right|_{s=-1} = \left. k_1 + k_2\frac{(s+1)}{s+2}\right|_{s=-1}$$

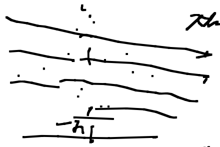
$$k_2 = \left.\frac{3s}{s+1}\right|_{s=-2} = \frac{-2\times3}{-2+1} = +2\times3 = 6$$

s-plane $\Rightarrow s = \sigma + j\omega$

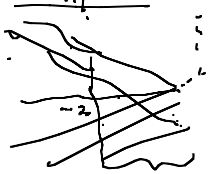


$$\mathcal{L}[e^{-at}1(t)] = \left.\frac{1}{s+a}\right|_{\sigma > -a}$$

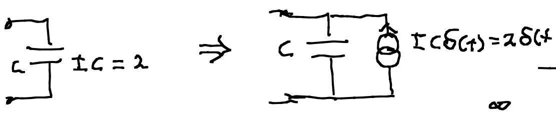
Then $f(t) = -3e^{-t}1(t) + 6e^{-2t}1(t)$; $\sigma > -1$



here $f(t) = 3e^{-t}1(-t) - 6e^{-2t}1(-t)$, $\sigma < -2$

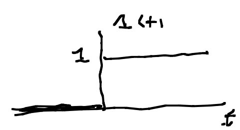


here $f(t) = 3e^{-t}1(-t) + 6e^{-2t}1(t)$, $-2 < \sigma < -1$



$\delta(t) \Rightarrow \int_{-\infty}^{\infty} g(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} g(0)\,\delta(t)\,dt = g(0)\int_{-\infty}^{\infty}\delta(t)\,dt = g_0 \cdot 1$

$1(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$; $\frac{d1(t)}{dt} = \delta(t)$



$T(s) = \frac{N(s)}{D(s)}$.

output $= \mathcal{L}^{-1}[T(s)]$

if the input is 1, ; $\frac{N(s)}{D(s)} \cdot \frac{1}{s}$

$\mathcal{L}[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)\,e^{-st}\,dt = e^{-s0}\int_{-\infty}^{\infty}\delta(t)\,dt = 1 \cdot 1$

$\mathcal{L}[1(t)] = \int_{-\infty}^{\infty} 1(t)\,e^{-st}\,dt = \int_{0}^{\infty} e^{-st} = -\frac{-1}{s} = \frac{1}{s}$

$\sigma > 0$

Ex: $y(s) = \frac{2s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)} = 2s + \underbrace{\frac{k_1}{s+j1} + \frac{k_1^*}{s-j1}}_{\frac{sk_1 - k_1 j1 + sk_1^* + jk_1^*}{s(2\mathrm{Re}\,k_1)} \;\; \frac{2k_1 s}{s^2+1}} + \underbrace{\frac{k_2}{s+j\sqrt{3}} + \frac{k_2^*}{s-j\sqrt{3}}}_{\frac{2k_2 s}{s^2+3}}$

$= 2s + \frac{2k_1 s}{s^2+1} + \frac{2k_2 s}{s^2+3}$

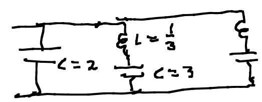
k_1 & k_2 are residues

$2k_1 s = (s^2+1)\left(\frac{2s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)}\right.$

$\left. - 2s(s^2+1) - \frac{2k_2 s}{s^2+3}\times(s^2+1)\right|_{s^2=-1} = -\left.\frac{2s(s^2+2)(s^2+4)}{s^2+3}\right|_{s^2=-1} + 0 + 0$

$$2k_1 \frac{s}{s} = +2\frac{s}{s}\left(\frac{(-1+2)(-1+4)}{(-1+3)}\right) = 2k_1 = +2\left(\frac{1\cdot 3}{2}\right) \Rightarrow k_1 = +\frac{3}{2}$$

$$Y(s) = 2s + \frac{+3s}{s^2+1} + \frac{2k_2 s}{s^2+3} \Rightarrow$$



degree 5 circuit
5 = degree

$$\Downarrow$$
$$Y_1$$
$$\Downarrow$$
$$Z_1 = \frac{s^2+1}{3s} = \frac{1}{3}s + \frac{1}{3s}$$

Singular value decomposition of A, $m \times n$ matrix

$A = U \Sigma V$, U is orthogonal, $UU^T = 1_m = U^T U$,
V is orthogonal $V^T V = 1_n = V V^T$
Σ is diagonal with non-negative entries
ordered higest to lowest, $m \times n$ matrix

From MathCad

subscript=[, transpose=Ctrl1
SVD = svd2( )

$$A := \begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 0 & 0 \end{pmatrix} \qquad A2 := \begin{pmatrix} 2 & -3 \\ 4 & 5 \\ 0 & 0 \end{pmatrix}$$

$$Suv := svd2(A2)$$

$$Suv = \begin{pmatrix} \{2,1\} \\ \{3,2\} \\ \{2,2\} \end{pmatrix} \quad Suv_0 = \begin{pmatrix} 6.531 \\ 3.369 \end{pmatrix} \quad Suv_1 = \begin{pmatrix} -0.23 & 0.973 \\ 0.973 & 0.23 \\ 0 & 0 \end{pmatrix} \quad Suv_2 = \begin{pmatrix} 0.526 & 0.851 \\ 0.851 & -0.526 \end{pmatrix}$$

$$Suv_1 \cdot diag(Suv_0) \cdot Suv_2 = \begin{pmatrix} 2 & -3 \\ 4 & 5 \\ 0 & 0 \end{pmatrix} \qquad Suv_2 \cdot {Suv_2}^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$${Suv_1}^T = \begin{pmatrix} -0.23 & 0.973 & 0 \\ 0.973 & 0.23 & 0 \end{pmatrix} \qquad {Suv_1}^T \cdot Suv_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$Suv_1 \cdot {Suv_1}^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$